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# $\gamma Z$ Pair Production at the Photon Linear Collider

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## Abstract

$\gamma\gamma \rightarrow \gamma Z$  scattering at the Photon Linear Collider is considered. Explicit formulas for helicity amplitudes due to  $W$  boson loops are presented. It is shown that the  $Z\gamma$  pair production will be easily observable at PLC and separation of the  $W$  loop contribution will be possible at  $e^+e^-$  c.m. energy of 300 GeV or higher.

# 1. Introduction

The coupling of the three photons to  $Z$ -boson is absent at the classical level and is generated only at the one loop order. The study of this coupling gives us a possibility to probe nonabelian structure of the Electroweak Interactions, through the  $W$  boson loop contribution involving both triple and quartic bosonic vertices, in addition to the fermionic loop diagrams.

Until recently a special attention was attracted by the decay  $Z \rightarrow \gamma\gamma\gamma$ , motivated by  $LEP$  experiments. The decay width was calculated taking into account both fermion [1] and  $W$  [2, 3] loop contributions. The conclusion is that the  $Z \rightarrow \gamma\gamma\gamma$  decay width is too small and is clearly beyond even the high luminosity option of  $LEP$ .

With the advent of the new collider technique [4] a more favorable opportunity for the experimental study of the  $\gamma\gamma\gamma Z$  coupling appears. It is the scattering reaction  $\gamma\gamma \rightarrow \gamma Z$  in the collision of high energy high intensity photon beams at the Photon Linear Collider (PLC).

The  $W$  loop contribution to the polarization tensor of the process  $Z \rightarrow \gamma\gamma\gamma$  was derived recently in [3], where the decay width was calculated. To minimize the number of the Feynman diagrams all calculations were done in unitary gauge, where the individual diagrams contain superficial divergencies which were canceled before reducing the tensor integrals to scalar integrals.

We have derived the helicity amplitudes for  $\gamma\gamma \rightarrow \gamma Z$  in 't Hooft-Feynman and non-linear gauges [5]. In both these gauges superficial divergencies do not appear. In 't Hooft-Feynman gauge there are much more diagrams due to the triple  $W$ -Nambu-Goldstone-photon ( $Z$  boson) couplings than in unitary gauge. However, in non-linear gauge, where such mixed coupling do not appear, there are just diagrams with either  $W$  boson, NG or ghost fields inside the loop.

In Section 2 we present explicit analytic expressions for the  $W$ -boson loop contribution to the helicity amplitudes. Section 3 contains the numerical results. It is shown that  $\gamma\gamma \rightarrow \gamma Z$  cross section is large enough to be observable at the PLC and even separation of the  $W$  loop should be possible at large energy. This fact has a fundamental significance for testing triple and quartic  $W$  boson vertices. Finally, in Section 4 conclusions are made.

## 2. Helicity Amplitudes

We use the reduction algorithm of [6] to express the helicity amplitudes of the reaction

$$\gamma(p_1)\gamma(p_2) \rightarrow \gamma(p_3)Z(p_4)$$

in terms of the set of basic scalar loop integrals. As noted above, all calculations were done in both 't Hooft-Feynman and non-linear gauges. The algebraic calculations were carried out using symbolic manipulation program FORM [7].

Leaving out the factor  $\alpha^2 \cos \theta_W / \sin \theta_W$ , we find the nine independent amplitudes:

$$\begin{aligned}
A_{++++}(s, t, u) = & \frac{16s_1}{s} E(t, u) + \\
& 4(2(s - 4m_W^2)s_1 - m_W^2(m_Z^2 - 6m_W^2))[D(s, t) + D(s, u) + D(t, u)] + \\
& 2\left(\frac{m_Z^2}{m_W^2} - 6\right) \left\{ \frac{tu + m_W^2(s_1 + s)}{ss_1} E(t, u) - \frac{2m_W^2}{s_1} [tuD(t, u) + m_Z^2 C(s)] - \right. \\
& \left. \frac{(s + m_Z^2)tu}{s_1 t_1 u_1} - \left[ \frac{2m_W^2 m_Z^2 s}{s_1 t_1} C_1(t) - \left( \frac{2t + s}{s_1} - \frac{m_Z^4 s}{s_1 t_1^2} \right) B_1(t) + (u \leftrightarrow t) \right] \right\}. \quad (2.1)
\end{aligned}$$

$$\begin{aligned}
A_{+++-}(s, t, u) = & 2\left(\frac{m_Z^2}{m_W^2} - 6\right) \left\{ -2m_W^4(D(s, t) + D(s, u) + D(t, u)) - \frac{m_Z^2 tu}{s^2 s_1} E(t, u) + \right. \\
& m_W^2 \left( \frac{4m_Z^2 - s}{ss_1} D(t, u) - \frac{s(u^2 + t^2)}{s_1 tu} C(s) - \frac{s_1^2}{ut} C_1(s) \right) + \frac{(s + m_Z^2)ut}{s_1 t_1 u_1} + \\
& \left[ m_W^2 \left( \left( \frac{(m_Z^2 u - st)s}{s_1 t_1 u} + \frac{2m_Z^2 u - su_1}{s_1 s} \right) C_1(t) - \frac{(2m_Z^2 u + st)t}{sus_1} C(t) - \right. \right. \\
& \left. \left. \frac{st}{u} D(s, t) \right) + \frac{m_Z^2(2t_1 - s)ut}{ss_1 t_1^2} B_1(t) + (u \leftrightarrow t) \right] \right\}. \quad (2.2)
\end{aligned}$$

$$\begin{aligned}
A_{++-+}(s, t, u) = & 2\left(\frac{m_Z^2}{m_W^2} - 6\right) \left\{ -2m_W^4[D(s, t) + D(s, u) + D(t, u)] - \right. \\
& m_W^2 \left( \frac{tu}{s_1} D(t, u) + \frac{s(u^2 + t^2)}{s_1 tu} C(s) + \frac{s_1^2}{ut} C_1(s) \right) + 1 - \\
& \left[ m_W^2 \left( \frac{st}{u} D(s, t) + \frac{u^2 + t^2}{s_1 u} C(t) + \frac{tt_1}{s_1 u} C_1(t) \right) + (u \leftrightarrow t) \right] \right\}. \quad (2.3)
\end{aligned}$$

$$\begin{aligned}
A_{+-+--}(s, t, u) = & 2\left(\frac{m_Z^2}{m_W^2} - 6\right) \left\{ 1 - 2m_W^4[D(s, t) + D(s, u) + D(t, u)] - \right. \\
& \left. \frac{m_W^2 m_Z^2}{ss_1} [E(u, t) + 2sC(s)] \right\}. \quad (2.4)
\end{aligned}$$

$$\begin{aligned}
A_{+--+}(s, t, u) = A_{-++-}(s, u, t) = & \frac{16s}{s_1} E(s, u) + \\
& 4\left(\frac{2st(t - 4m_W^2)}{s_1} - m_W^2(m_Z^2 - 6m_W^2)\right)[D(s, t) + D(s, u) + D(t, u)] +
\end{aligned}$$

$$2\left(\frac{m_Z^2}{m_W^2} - 6\right)\left\{\left(\frac{su}{t^2} + \frac{2m_W^2}{t}\right)E(s, u) - m_W^2\left(\frac{2su}{t}D(s, u) + \frac{m_Z^2}{ss_1}E(t, u) + \frac{2m_Z^2(2u_1 + s)}{s_1u_1}C_1(u)\right) + \frac{s(s_1 - u)}{s_1t}B_1(s) - \frac{su(2s_1u_1 + tu)}{s_1tu_1^2}B_1(u) - \frac{su}{s_1u_1}\right\} \quad (2.5)$$

$$A_{+---}(s, t, u) = A_{-++}(s, u, t) = \frac{8m_Z^2t}{s_1u}(2E(s, t) - u(4m_W^2 - u)(D(s, t) + D(s, u) + D(t, u)) - 2\left(\frac{m_Z^2}{m_W^2} - 6\right)\left\{m_W^2\left[\left(\frac{st}{u} + 2m_W^2\right)D(s, t) + \left(\frac{su}{t} + 2m_W^2\right)D(s, u) + \left(\frac{tu}{s_1} + 2m_W^2\right)D(t, u) + \frac{ss_1}{ut}C(s) + \frac{t^2}{s_1u}C(t) + \frac{t^2 + s_1^2}{s_1t}C(u) + \frac{u^2 + t^2}{ut}C_1(s) + \frac{2m_Z^2u^2 + t_1(uu_1 + ss_1)}{s_1t_1u}C_1(t) + \frac{uu_1}{s_1t}C_1(u)\right] + \frac{m_Z^2tu}{s_1t_1^2}B_1(t) - \frac{st}{s_1t_1}\right\} \quad (2.6)$$

$$A_{+++0}(s, t, u)/(p_t/\sqrt{2}) = 2m_Z\left(\frac{m_Z^2}{m_W^2} - 6\right)\left\{(t - u)\left(\frac{3m_W^2}{s_1}D(t, u) - \frac{E(t, u)}{ss_1} + \frac{2m_W^2s}{s_1tu}C(s) + \frac{2s}{s_1t_1u_1}\right) + \left[m_W^2\left(\frac{s}{u}D(s, t) + \frac{2}{s_1}C(t) + 2\frac{2s^2 - t_1^2}{s_1t_1u}C_1(t)\right) + \frac{2t_1t + m_Z^2u}{s_1t_1^2}B_1(t) - (u \leftrightarrow t)\right]\right\}. \quad (2.7)$$

$$A_{++-0}(s, t, u)/(p_t/\sqrt{2}) = 2m_Z(m_Z^2 - 6m_W^2)\left\{\frac{(t - u)}{s_1}\left(D(t, u) - \frac{2s}{ut}\right) - \left[\frac{s}{u}D(s, t) - \frac{2}{s_1}C(t) + \frac{2t_1}{s_1u}C_1(t) - (u \leftrightarrow t)\right]\right\}. \quad (2.8)$$

$$A_{+-+0}(s, t, u)/(p_t/\sqrt{2}) = A_{+--0}(s, u, t)/(p_t/\sqrt{2}) = -\frac{16m_Zs}{s_1}\left((t - 4m_W^2)(D(s, t) + D(s, u) + D(t, u)) + \frac{2E(s, u)}{t}\right) - 2m_Z\left(\frac{m_Z^2}{m_W^2} - 6\right)\left\{m_W^2\left(\frac{(u - t)}{s_1}D(t, u) + \frac{s}{u}D(s, t) + \frac{3s}{t}D(s, u)\right) + \right.$$

$$\begin{aligned} & \frac{2m_W^2}{s_1} \left( C(u) - C(t) - \frac{ss_1}{ut} C(s) + \frac{t_1}{u} C_1(t) + \frac{u_1}{t} C_1(u) - \frac{2t}{u_1} C_1(u) \right) \\ & - \frac{s}{t^2} E(s, u) + \frac{2s}{s_1 t} B_1(s) - 2 \left( \frac{1}{t} + \frac{m_Z^2 t}{s_1 u_1^2} \right) B_1(u) + \frac{2s}{s_1 u_1} \Big\}. \end{aligned} \quad (2.9)$$

The other helicity amplitudes can be obtained by parity transformation. We define

$$s = (p_1 + p_2)^2, \quad t = (p_2 + p_3)^2, \quad u = (p_1 + p_3)^2.$$

All amplitudes with longitudinal polarization of  $Z$ -boson contain  $p_t$  – transverse momentum of final particles

$$p_t = \sqrt{\frac{tu}{s}}.$$

The scalar three-point functions are given by

$$C(s) = \frac{1}{i\pi^2} \int \frac{d^4 q}{(q^2 - m_W^2) ((q + p_1)^2 - m_W^2) ((q + p_1 + p_2)^2 - m_W^2)}, \dots \quad (2.10)$$

and other two-, three- and four-point functions  $B$ ,  $C$  and  $D$  are defined by analogous expressions. Only the following combination of the two-point functions is present:

$$B_1(s) = B(s) - B(m_Z^2).$$

$C_1(s)$  is the three-point function with one external massless and another massive lines, *e.g.*  $p_3^2 = 0$ ,  $p_4^2 = m_Z^2$ ,  $(p_3 + p_4)^2 = s$ . The expressions of the scalar loop functions in terms of Spence functions are known [8].  $E(s, t)$  is the auxiliary function defined as:

$$E(s, t) = sC(s) + tC(t) + s_1 C_1(s) + t_1 C_1(t) - stD(s, t).$$

After routine manipulations helicity amplitudes expressed through the  $\gamma Z \gamma \gamma$  polarization tensor obtained in [3] can be reduced analytically to our expressions (1)-(9).

### 3. Numerical Results

We first present different contributions to the polarized cross sections for  $\gamma Z$ -pair production in monochromatic  $\gamma \gamma$  collisions. We consider the extreme cases of  $\lambda_{\gamma_1} \lambda_{\gamma_2} = \pm 1$ , i.e. full circular polarization for the incoming photons. The cross section is given by

$$\frac{d\sigma_{\lambda_1 \lambda_2}(s)}{d\cos\theta} = \sum_{\lambda_3, \lambda_4} \frac{\alpha^4 \cos^2 \theta_W}{32\pi s \sin^2 \theta_W} |A_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}|^2 \left(1 - \frac{m_Z^2}{s}\right).$$

We used the following numerical values of parameters  $\alpha = 1/128$ ,  $m_W = 80.22$  GeV,  $m_Z = 91.173$  GeV. Contribution of fermionic loops is calculated using the expressions for helicity amplitudes from [1]. For the masses of fermions we take the following

values  $m_u = m_d = m_s = m_c = m_e = m_\mu = m_\tau = 100$  MeV,  $m_b = 5$  GeV. As is shown in Figs. 1a,b  $W$  loop contribution dominates at photon-photon collision energies above 250 GeV. Cross section for equal initial photon helicities  $\lambda_{\gamma_1}\lambda_{\gamma_2} = 1$  is about two times larger than that for opposite photon helicities  $\lambda_{\gamma_1}\lambda_{\gamma_2} = -1$ . Also the cross section of transverse  $Z_T$  boson production is at least two orders of magnitude larger than that for longitudinal  $Z_L$  boson production. Asymptotically cross section of  $\gamma Z_T$  production in photon fusion is about 7 times larger than the cross section of photon-photon scattering  $\gamma\gamma \rightarrow \gamma\gamma$ , calculated earlier [9]. The reason is that  $ZWW$  coupling is  $\cos\theta_W/\sin\theta_W$  times larger than the photon  $WW$  one, and a symmetry factor of 1/2 is missing.

For calculating the realistic cross sections of  $\gamma\gamma \rightarrow \gamma Z$  in PLC we assume that 90% electron (positron) beam longitudinal polarization ( $\lambda_{e_{1,2}} = \pm 0.45$ ) and 100% laser beam circular polarization ( $\lambda_{\gamma_{1,2}} = \pm 1$ ) are achievable. We consider the following polarizations of electron and laser beams:  $2\lambda_{e_1} = 2\lambda_{e_2} = 0.9$  and  $\lambda_{\gamma_1} = \lambda_{\gamma_2} = -1$ , which give the photon-photon energy spectrum peaking just below the highest allowed photon-photon energy with mostly equal photon helicities [4]. Based on the  $e^+e^-$  linear collider, PLC will have almost the same energy and luminosity, i.e. c.m. energy of 100 – 500 GeV and luminosity of order  $10^{33}\text{cm}^{-2}\text{s}^{-1}$  are considered [4]. In Fig. 2 are presented cross sections for  $\gamma\gamma \rightarrow \gamma Z$  scattering for longitudinal and transversal polarization of  $Z$  boson as a function of the  $e^+e^-$  c.m. energy for polarized initial electron and laser beams. We restrict  $Z$  boson scattering angle,  $|\cos\theta_Z| < \cos 30^\circ$ . The contributions of  $W$  and fermionic loops as well as their sum are given separately. Also a background from resolved photon contribution  $q\bar{q} \rightarrow \gamma Z$  is shown. We used the parametrization of photonic parton distributions from [10]. As in Fig. 1, the cross section for transversely polarized  $Z$  boson is two orders of magnitude larger than longitudinal one. The resolved photon contribution is larger than longitudinal  $Z_L$  boson contribution, but more than an order of magnitude smaller than dominating transverse  $Z_T$  boson contribution. At  $e^+e^-$  collision energies above 250 GeV  $W$ -loop contribution dominates. At  $\sqrt{s_{e^+e^-}} = 300$  GeV  $\sigma_{\gamma\gamma \rightarrow \gamma Z} = 10$  fb, while at  $\sqrt{s_{e^+e^-}} = 500$  GeV  $\sigma_{\gamma\gamma \rightarrow \gamma Z} = 50$  fb, which corresponds to 100-500 events for integrated luminosity of  $\int \mathcal{L} dt = 10 \text{ fb}^{-1}$ . Unlike the case of hadron collider, PLC provides an attractive environment not only for leptonic  $Z \rightarrow ee, \mu\mu$ , but also for hadronic decay modes  $Z \rightarrow q\bar{q}$ .  $\pi^0$  background events in jets  $\gamma\gamma \rightarrow q\bar{q}Z$  and continuum  $\gamma\gamma \rightarrow q\bar{q}\gamma$  production must be rejected by good geometric resolution and stringent isolation criteria combined with fair jet-jet hadronic energy resolution to detect the  $Z$  peak and accompanying photon. The detection of both photon and  $Z$  boson is also necessary to suppress the background from the photon scattering off the residual electrons left over from the original Compton backscattering  $\gamma e \rightarrow Ze$ .

Using the helicity amplitudes from Section 3 we calculated the  $Z \rightarrow \gamma\gamma\gamma$  decay rate. The  $W$  loop contribution of 0.026 eV coincides with the calculation of [3]. For the total width we obtained 1.5 eV, which is larger than calculated in [3]. However this difference is due to another value for  $W$  mass used in [3], because the fermionic

contribution to decay rate does depend sensitively on the choice of  $\sin\theta_W^2 = 1 - m_W^2/m_Z^2$ .

## 4. Conclusions

We obtained the compact expressions for  $W$  loop contributions to the helicity amplitudes for  $Z\gamma\gamma\gamma$  scattering in two different gauges ('t Hooft-Feynman and non-linear ones). As a result, we calculated cross sections of  $\gamma\gamma \rightarrow \gamma Z$  scattering at the Photon Linear Collider. Numerical values of the cross sections indicate, that this process should be easily observable at the PLC. For  $e^+e^-$  c.m. energy of 300 – 500 GeV or higher it will be possible to separate the  $W$  boson loop contribution. Taking into account that  $\gamma Z$  final state should be background free, we conclude that hundreds of  $\gamma Z$  pair production events yearly should be observable at PLC. This fact will have fundamental significance, because this process is a pure one-loop effect of the Standard Model as a renormalizable nonabelian gauge theory. As such, the observation of this reaction will provide a possibility to test triple and quartic  $\gamma WW$ ,  $ZWW$ ,  $\gamma\gamma WW$  and  $\gamma ZWW$  vertices. If the anomalous triple vertex, contributing mainly for transverse gauge bosons, *e.g.* so called blind direction operator  $Tr(W_{\alpha\beta}W^{\beta\gamma}W_{\gamma}^{\alpha})$  [11], exists its effect can be probed in the reaction  $\gamma\gamma \rightarrow \gamma Z$ . For a composite  $Z$  boson [12] one can also expect to observe a deviation from the Standard Model predictions.

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## Figure captions

Fig. 1. Total cross section of  $\gamma Z$  pair production in monochromatic photon-photon collisions versus  $\gamma\gamma$  c.m. energy for different helicities of the incoming photons and final  $Z$  boson. Total cross section (solid line) as well as  $W$  boson loop contribution (dashed line) and fermion loop contribution (dotted line) are shown.

Fig. 2. Cross section of  $\gamma Z$  pair production in  $\gamma\gamma$  collisions versus c.m. energy of the  $e^+e^-$  collisions computed taking into account photon spectrum of the backscattered laser beams. Dash-dotted line shows the resolved photon contribution. The other notations are the same as in Fig. 1.

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